

ΑΠΑΝΤΗΣΕΙΣ:
ΑΛΓΕΒΡΑ Β' ΛΥΚΕΙΟΥ 1/2/2015
ΕΞΕΤΑΖΟΜΕΝΗ ΥΛΗ ΤΡΙΓΩΝΟΜΕΤΡΙΑ

ΘΕΜΑ Α

A₁. Σχολικό βιβλίο σελίδα 74.

A₂. Σχολικό βιβλίο σελίδα 60.

A₃. α-Σ, β-Σ, γ-Σ, δ-Λ, ε-Λ.

ΘΕΜΑ Β

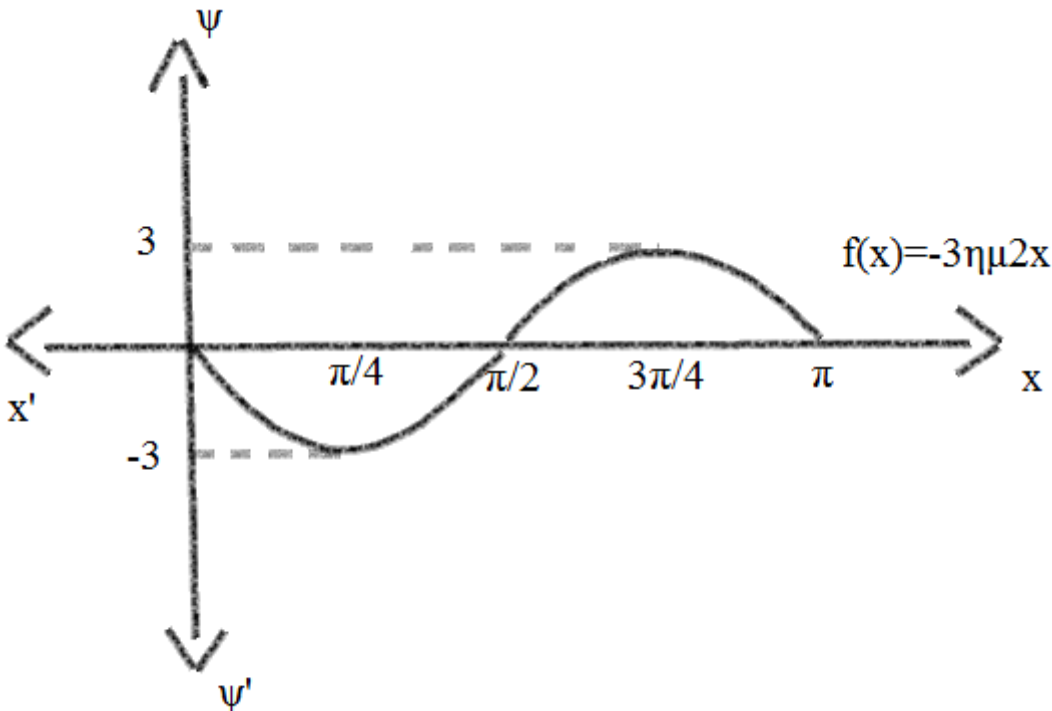
B₁. $T = \frac{2\pi}{\omega} = \pi \Rightarrow \omega = \frac{2\pi}{\pi} = 2.$

$\max f(x) = |\rho| = 3$ για $\rho < 0$ έχουμε $\rho = -3.$

B₂. $f(x) = -3\eta\mu 2x$

$\max f(x) = 3, \min f(x) = -3$ και $T = \pi.$

$\frac{T}{4} = \frac{\pi}{4}, \frac{T}{2} = \frac{\pi}{2}, \frac{3T}{4} = \frac{3\pi}{4}, T = \pi.$



$$B_3. f\left(\frac{x}{2}\right) + f\left(x - \frac{\pi}{6}\right) = 0 \Rightarrow -3\eta\mu 2\left(\frac{x}{2}\right) - 3\eta\mu\left[2\left(x - \frac{\pi}{6}\right)\right] = 0 \Rightarrow$$

$$-3\eta\mu x - 3\eta\mu\left(2x - \frac{\pi}{3}\right) = 0 \Rightarrow -\eta\mu x = \eta\mu\left(2x - \frac{\pi}{3}\right) \Rightarrow$$

$$\eta\mu(-x) = \eta\mu\left(2x - \frac{\pi}{3}\right) \text{ άρα } \left. \begin{array}{l} -x = 2\kappa\pi + 2x - \frac{\pi}{3} \\ \text{ή} \\ -x = 2\kappa\pi + \pi - \left(2x - \frac{\pi}{3}\right) \end{array} \right\} \kappa \in \mathbb{Z}$$

$$\left. \begin{array}{l} -3x = 2\kappa\pi - \frac{\pi}{3} \\ \text{ή} \\ x = 2\kappa\pi + \pi + \frac{\pi}{3} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{-2\kappa\pi}{3} + \frac{\pi}{9} \\ x = 2\kappa\pi + \frac{4\pi}{3} \end{array} \right\} \kappa \in \mathbb{Z}$$

ΘΕΜΑ Γ

$$\Gamma_1. 2\eta\mu^2(\pi + x) + \epsilon\varphi\left(\frac{\pi}{2} - x\right) \cdot \epsilon\varphi(2\pi - x) + 2\eta\mu^2\left(\frac{\pi}{2} - x\right) =$$

$$2(-\eta\mu x)^2 + \sigma\varphi x \cdot \epsilon\varphi(-x) + 2\sigma\upsilon\nu^2 x = 2\eta\mu^2 x - \sigma\varphi x \cdot \epsilon\varphi x + 2\sigma\upsilon\nu^2 x =$$

$$2(\eta\mu^2 x + \sigma\upsilon\nu^2 x) - 1 = 2 \cdot 1 - 1 = 1.$$

$$\Gamma_2. f(4\pi - x) = \epsilon\varphi(4\pi - x) = \epsilon\varphi(-x) = -\epsilon\varphi x$$

$$f\left(\frac{3\pi}{2} - x\right) = \epsilon\varphi\left(\frac{3\pi}{2} - x\right) = \epsilon\varphi\left(\pi + \frac{\pi}{2} - x\right) = \epsilon\varphi\left(\frac{\pi}{2} - x\right) = \sigma\varphi x$$

$$f\left(\frac{7\pi}{2} + x\right) = \epsilon\varphi\left(\frac{7\pi}{2} + x\right) = \epsilon\varphi\left(3\pi + \frac{\pi}{2} + x\right) = \epsilon\varphi\left(\pi + \frac{\pi}{2} + x\right) =$$

$$\epsilon\varphi\left(\frac{\pi}{2} + x\right) = \epsilon\varphi\left(\frac{\pi}{2} - (-x)\right) = \sigma\varphi(-x) = -\sigma\varphi x$$

$$f(5\pi + x) = \epsilon\varphi(5\pi + x) = \epsilon\varphi(\pi + x) = \epsilon\varphi x$$

$$\eta\mu^2(3\pi + x) = \eta\mu^2(\pi + x) = (-\eta\mu x)^2 = \eta\mu^2 x$$

$$\sigma\upsilon\nu^2(5\pi - x) = \sigma\upsilon\nu^2(\pi - x) = (-\sigma\upsilon\nu x)^2 = \sigma\upsilon\nu^2 x$$

$$\text{Άρα } A = \frac{-\epsilon\phi x \cdot \sigma\phi x \cdot (-\sigma\phi x) \cdot \epsilon\phi x}{\eta\mu^2 x + \sigma\upsilon\nu^2 x} = \frac{(-1) \cdot (-1)}{1} = 1$$

$$\begin{aligned} \Gamma_3 \alpha) \frac{\sigma\upsilon\nu\theta}{1-\eta\mu\theta} + \frac{\sigma\upsilon\nu\theta}{1+\eta\mu\theta} &= \frac{\sigma\upsilon\nu\theta(1+\eta\mu\theta)}{(1-\eta\mu\theta)(1+\eta\mu\theta)} + \frac{\sigma\upsilon\nu\theta(1-\eta\mu\theta)}{(1+\eta\mu\theta)(1-\eta\mu\theta)} = \\ \frac{\sigma\upsilon\nu\theta + \sigma\upsilon\nu\theta \cdot \eta\mu\theta + \sigma\upsilon\nu\theta - \sigma\upsilon\nu\theta \cdot \eta\mu\theta}{(1-\eta\mu\theta)(1+\eta\mu\theta)} &= \frac{2\sigma\upsilon\nu\theta}{1-\eta\mu^2\theta} = \frac{2\sigma\upsilon\nu\theta}{\sigma\upsilon\nu^2\theta} = \frac{2}{\sigma\upsilon\nu\theta}. \end{aligned}$$

β) Περιορισμοί:

$$1 - \eta\mu\theta \neq 0 \Rightarrow \eta\mu\theta \neq 1 \Rightarrow \eta\mu\theta \neq \eta\mu \frac{\pi}{2} \Rightarrow \theta \neq 2\kappa\pi + \frac{\pi}{2}$$

$$1 + \eta\mu\theta \neq 0 \Rightarrow \eta\mu\theta \neq -1 \Rightarrow \eta\mu\theta \neq \eta\mu(-\frac{\pi}{2}) \Rightarrow \theta \neq 2\kappa\pi - \frac{\pi}{2}$$

$$\sigma\upsilon\nu\theta \neq 0 \Rightarrow \sigma\upsilon\nu\theta \neq \sigma\upsilon\nu \frac{\pi}{2} \Rightarrow \theta \neq 2\kappa\pi \pm \frac{\pi}{2}$$

$$\text{άρα πρέπει } \theta \neq \kappa\pi + \frac{\pi}{2}$$

Οπότε έχουμε:

$$\frac{\sigma\upsilon\nu\theta}{1-\eta\mu\theta} + \frac{\sigma\upsilon\nu\theta}{1+\eta\mu\theta} = 2 \text{ όμως από το } \Gamma_2 \alpha \text{ έχω}$$

$$\frac{2}{\sigma\upsilon\nu\theta} = 2 \Rightarrow \sigma\upsilon\nu\theta = 1 \Rightarrow \sigma\upsilon\nu\theta = \sigma\upsilon\nu 0 \text{ άρα } 0 \leq \theta < \pi \Rightarrow 0 \leq 2\kappa\pi < \pi$$

$$\Rightarrow 0 \leq \kappa < \frac{1}{2}, \kappa \in \mathbb{Z} \text{ άρα } \kappa = 0, \text{ οπότε } \theta = 0.$$

ΘΕΜΑ Δ

$$\Delta_1. \sin 3x = \sin(2x + x) = \sin 2x \cdot \sin x - \eta\mu 2x \cdot \eta\mu x = (2\sin^2 x - 1) \cdot \sin x - (2\eta\mu x \cdot \sin x) \cdot \eta\mu x =$$

$$2\sin^3 x - \sin x - 2\eta\mu^2 x \cdot \sin x = 2\sin^3 x - \sin x - 2(1 - \sin^2 x) \cdot \sin x =$$

$$2\sin^3 x - \sin x - 2\sin x + 2\sin^3 x = 4\sin^3 x - 3\sin x, \forall x \in \mathbb{R}$$

$$\Delta_2. \alpha + \beta = \frac{\pi}{4} \Rightarrow \beta = \frac{\pi}{4} - \alpha, \text{ άρα}$$

$$\sigma\phi\beta = \sigma\phi\left(\frac{\pi}{4} - \alpha\right) = \frac{\sigma\phi\frac{\pi}{4} \cdot \sigma\phi\alpha + 1}{\sigma\phi\alpha - \sigma\phi\frac{\pi}{4}} = \frac{\sigma\phi\alpha + 1}{\sigma\phi\alpha - 1} \text{ οπότε}$$

$$(1 - \sigma\phi\alpha) \cdot (1 - \sigma\phi\beta) = (1 - \sigma\phi\alpha) \left(1 - \frac{\sigma\phi\alpha + 1}{\sigma\phi\alpha - 1}\right) = (1 - \sigma\phi\alpha) \cdot \left(\frac{\sigma\phi\alpha - 1 - \sigma\phi\alpha - 1}{\sigma\phi\alpha - 1}\right) =$$

$$\frac{-2(1 - \sigma\phi\alpha)}{\sigma\phi\alpha - 1} = 2.$$

$$\Delta_3. \sin 2x + 10\eta\mu^2 \frac{x}{2} - 2 = 0 \Rightarrow (2\sin^2 x - 1) + 10 \cdot \frac{1 - \sin 2\left(\frac{x}{2}\right)}{2} - 2 = 0 \Rightarrow$$

$$2\sin^2 x - 1 + 5 \cdot (1 - \sin x) - 2 = 0 \Rightarrow 2\sin^2 x - 1 + 5 - 5\sin x - 2 = 0 \Rightarrow$$

$$2\sin^2 x - 5\sin x + 2 = 0, \text{ θέτω } \sin x = \alpha \text{ και έχω}$$

$$2\alpha^2 - 5\alpha + 2 = 0$$

$$\Delta = 25 - 4 \cdot 2 \cdot 2 = 9$$

$$\alpha_{1,2} = \frac{5 \pm 3}{4} = 2 \text{ ή } \frac{1}{2}.$$

$$\text{άρα } \sin x = 2 \text{ όπου αππορίπτεται γιατί } \sin x \leq 1 \text{ ή } \sin x = \frac{1}{2}.$$

$$\text{Και τελικά έχουμε } \sin x = \frac{1}{2} \Rightarrow \sin x = \sin \frac{\pi}{3} \Rightarrow$$

$$x = 2\kappa\pi \pm \frac{\pi}{3}, \kappa \in \mathbb{Z}$$

**ΤΙΣ ΑΠΑΝΤΗΣΕΙΣ ΤΟΥ ΔΙΑΓΩΝΙΣΜΑΤΟΣ
ΕΠΙΜΕΛΗΘΗΚΑΝ ΟΙ ΚΑΘΗΓΗΤΕΣ:
ΓΑΣΠΑΡΑΤΟΣ ΑΝΔΡΕΑΣ
ΙΜΠΟΣ ΧΡΗΣΤΟΣ
ΝΙΚΟΥ ΔΗΜΗΤΡΗΣ
ΝΤΑΝΤΙΝΟΣ ΓΙΩΡΓΟΣ
ΠΑΥΛΟΥ ΝΙΚΟΣ**

SYSTHMA